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Minimizing Hiring Cost for Three Stage Flowshop Scheduling for a Fixed Sequence of Jobs

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ABSTRACT
The present paper pertains to a three stage flow shop scheduling to minimize the hiring cost of machines for a fixed sequence of jobs with a given makespan. The objective of the paper is to develop algorithm to obtain the latest times at which machines should be hired so as to minimize the total hiring cost without altering the makespan for a fixed sequence of jobs processing. The proposed algorithm provides an intuitive eye to the decision makers when to hire machines. A numerical illustration is also given to substantiate the algorithm.
Keywords: *Rental Policy, Idle time, Latest time, Hiring cost, Makespan.*

1. Introduction

In flow shop scheduling problems, the objective is to obtain a sequence of jobs which when processed on the machine will optimize some well defined criteria. The number of possible schedules in a flow shop scheduling problem involving n-jobs and m-machines are $(n!)^m$. Every job will go on these machines in a fixed order of machines. Early research on flow shop problems is based mainly on Johnson's [8], which give a procedure for finding an optimal solution for two machines or three machines with certain characteristics. The research in to flow shop scheduling has drawn a great attention in the last decade with the aim to increase the effectiveness of industrial production. Now a days, the decision makers for the manufacturing plant must find a way to successfully manage resources in order to produce products in the most efficient way with minimum hiring cost, when the machines are hired for a fixed required sequence of jobs processing without violating total elapsed time. Ignall and Scharge [7] introduced branch and bound technique in flow shop scheduling problems. Bagga, P.C. [2] discussed sequencing in rental situations. Szware [13] studied some special cases of the flowshop scheduling. Some of the noteworthy approaches are due to Gupta, J.N.D. [4], Yoshida and Hitomi [[15], Singh T.P. [13], Chandra Shekhran *et.al.* [3], Singh T.P. and Gupta, D. [5] and Narain [12] etc. Singh T.P., Gupta, D. and Sharma, s. [6] studied *n* x 2 general flow shop problem to minimize rental cost under a pre-defined rental policy in which the probabilities have been associated with processing time on each machine. We have extended the study made by Gupta and Sharma [6] by introducing the concept of minimizing the hiring cost without altering the total elapsed time for a fixed sequence of jobs processing.

2. Practical Situation

Many applied and experimental situations exist in our day-to-day working in factories and industrial production concerns etc where different jobs are processed on various machines. These jobs are required to process in a machine shop A, B, C, ---- in a specified order. Further, various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. For example, In garment industry, we find a industrialist who is new in industry does not buy expensive machines for roving, spinning, yarn manufacturing, weaving, dying equipments, etc., but instead takes on rent. Hiring of the equipment is an affordable and quick solution for upcoming industrialists which are presently constrained by the availability of limited funds due to the recent global economic recession. Hiring enables saving working capital, gives option for having the equipment, and allows up gradation to new technology.

3. Notations & Definitions

The various notations used in the present work are as follows:

S : Given fixed sequence of jobs
M_j : Machine j, j= 1, 2, 3
$a_{i,j}$: Processing time of i^{th} job on machine M_j
$p_{i,j}$: Probability associated to the processing time $a_{i,j}$
$A_{i,j}$: Expected processing time of i^{th} job on machine M_j
$t_{i,j}(S)$: Completion time of i^{th} job of sequence S on machine M_j

$I_{i,j}(S)$: Idle time of machine M_j for job i in the sequence S
$L_j(S)$: The latest time when machine M_j is taken on rent for sequence S
$t'_{i,j}(S)$: Completion time of i^{th} job of sequence S on machine M_j when machine M_j start processing jobs at time $L_j(S)$
$U_j(S)$: Utilization time for which machine M_j, when M_j starts processing jobs at time $L_j(S)$
C_j : Hiring cost per unit time of machine j
$H(S)$: Total hiring cost for the sequence S of all machine

3.1 Definition

Completion time of i^{th} job on machine M_j is denoted by $t_{i,j}$ and is defined as:

$t_{i,j} = max\ (t_{i-1,j}\ ,\ t_{i,j-1}) + a_{i,j} \times p_{i,j}$ for $j \geq 2$.

$= max\ (t_{i-1,j}\ ,\ t_{i,j-1}) + A_{i,j}$, where $A_{i,j}$= expected processing time of i^{th} job on machine j.

3.2 Definition

Completion time of i^{th} job on machine M_j when M_j starts processing jobs at time L_j is denoted by $t'_{i,j}$ and is defined as , $t'_{i,j} = L_j + \sum_{k=1}^{i} A_{k,j} = \sum_{k=1}^{i} I_{k,j} + \sum_{k=1}^{i} A_{k,j}$.

Also, $t'_{i,j} = \max(t_{i,j-1}, t'_{i-1,j}) + A_{i,j}$.

3.3. Hiring Policy

The machines will be hired as and when they are required and are returned as and when they are no longer required. .i.e. the first machine will be hired in the starting of the processing the jobs, 2nd and 3rd machines will be hired at their latest time and are returned as soon as the last job is completed on them.

3.4. Assumptions

1. No machine processes more than one job at a time.
2. Pre-emption of jobs are not allowed.
3. Machines never breakdown during the scheduling process.
4. Each job is processed through each of the machines once and only once.
5. All the jobs and the machines are available at the beginning of the processing.
6. Jobs are independent of each other.

4. Theorems

In support of the algorithm, the following theorems have been established to find the latest time at which machines should be hired so as to minimize the total hiring cost without altering the total elapsed time.

4.1. Theorem: *If machine M_3 start processing at the time $L_3 = \sum_{i=1}^{n} I_{i,3}$ then $t_{n,3}$ will remain unaltered .*

Proof: Let $t'_{i,3}$ be the competition time of i^{th} job on machine M_3 when M_3 starts processing of jobs at time L_3. We shall prove the theorem with the help of mathematical induction.

Let $P(n): t'_{n,3} = t_{n,3}$

Basic Step: For n = 1; $t'_{1,3} = L_3 + A_{1,3} = I_{1,3} + A_{1,3} = (A_{1,1}+A_{1,2})+A_{1,3} = t_{1,3}$.

Therefore, P (1) is true.

Induction Step: Let P (m) be true. .i.e. $t'_{m,3} = t_{m,3}$

Now, we shall show that P (m+1) is also true, i.e. $t'_{m+1,3} = t_{m+1,3}$

By definition; $t'_{m+1,3} = \max(t_{m+1,2}, t'_{m,3}) + A_{m+1,3}$

$$\therefore t'_{m+1,3} = \max(t_{m+1,2}, L_3 + \sum_{i=1}^{m} A_{i,3}) + A_{m+1,3} = \max(t_{m+1,2}, \sum_{i=1}^{m+1} I_{i,3} + \sum_{i=1}^{m} A_{i,3}) + A_{m+1,3}$$

$$= \max(t_{m+1,2}, \sum_{i=1}^{m} I_{i,3} + \sum_{i=1}^{m} A_{i,3} + I_{m+1,3}) + A_{m+1,3} = \max(t_{m+1,2}, t_{m,3} + I_{m+1,3}) + A_{m+1,3}$$

$$= \max\left(t_{m+1,2}, t_{m,3} + \max\left(\left(t_{m+1,2} - t_{m,3}\right), 0\right)\right) + A_{m+1,3} = \max\left(t_{m+1,2}, t'_{m,3}\right) + A_{m+1,3}$$

$$= \max\left(t_{m+1,2}, t_{m,3}\right) + A_{m+1,3} \text{ (By assumption)}$$

$$= t_{m+1,3}$$

Therefore, P(m+*1*) is true . Hence, by the principle of mathematical induction the result is true for all natural number n. Therefore the total elapsed time on 3rd machine $t_{n,3}$ will remain unaltered if machine M_3 start processing at the time $L_3 = \sum_{i=1}^{n} I_{i,3}$.

4.2. Lemma: *If M_3 starts processing jobs at $L_3 = \sum_{i=1}^{n} I_{i,3}$ then*

(i). $L_3 > t_{1,2}$

(ii). $t'_{k+1,3} \geq t_{k,2}$, $k > 1$.

Proof: $L_3 = \sum_{i=1}^{n} I_{i,3} = I_{1,3} + \sum_{i=2}^{n} I_{i,3} = t_{1,2} + \sum_{i=2}^{n} I_{i,3}$

Since $\sum_{i=2}^{n} I_{i,3} \geq 0 \Rightarrow L_3 \geq t_{1,2}$

Now, $I_{k,3} = max\{t_{k,2} - t_{k-1}, 0\}$

$I_{k,3} \geq t_{k,2} - t_{k-1}$, .i.e. $t_{k-1} + I_{k,3} \geq t_{k,2}$

$$\sum_{i=1}^{k-1} I_{i,3} + \sum_{i=1}^{k-1} A_{i,3} + I_{k,3} \geq t_{k,2}$$

$$\sum_{i=1}^{k} I_{i,3} + \sum_{i=1}^{k-1} A_{i,3} \geq t_{k,2}, \text{ But } \sum_{i=k+1}^{n} I_{i,3} \geq 0$$

$$\sum_{i=1}^{k} I_{i,3} + \sum_{i=k+1}^{n} I_{i,3} + \sum_{i=1}^{k-1} A_{i,3} \geq t_{k,2}$$

$$\sum_{i=1}^{n} I_{i,3} + \sum_{i=1}^{k-1} A_{i,3} \geq t_{k,2} \Rightarrow L_3 + \sum_{i=1}^{k-1} A_{i,3} \geq t_{k,2} \Rightarrow t'_{k-1,3} \geq t_{k,2}$$

Hence the lemma is proved.

4.3. Theorem: *The processing of jobs on* M$_2$ *at time* $L_2 = \min_{i \leq k \leq n} \{Y_k\}$ *keeps total elapsed time unaltered where*

$Y_1 = L_3 - A_{1,2}$ *and* $Y_k = t'_{k-1,3} - \sum_{i=1}^{k} A_{i,2}; k > 1$.

Proof: Since $L_2 = \min_{i \leq k \leq n} \{Y_k\} = Y_r$ (say)

In particular for k =1 ; $Y_r \leq Y_1$

$$\Rightarrow Y_r + A_{1,2} \leq Y_1 + A_{1,2}$$

$$\Rightarrow Y_r + A_{1,2} \leq L_3 \qquad \text{----- (1)} \quad \left(\because Y_1 = L_3 - A_{1,2}\right)$$

By Lemma 3.2; we have

$$t_{1,2} \leq L_3 \qquad \text{---- (2)}$$

Also, $t'_{1,2} = \max\left(Y_r + A_{1,2}, t_{1,2}\right)$

On combining, we get $t'_{1,2} \leq L_3$

For k >1, As $Y_r = \min_{i \leq k \leq n} \{Y_k\}$

$$\Rightarrow Y_r \leq Y_k ; \qquad k = 2, 3 \ldots\ldots\ldots, n$$

$$\Rightarrow Y_r + \sum_{i=1}^{n} A_{i,2} \leq Y_k + \sum_{i=1}^{n} A_{i,2}$$

$$\Rightarrow Y_r + \sum_{i=1}^{n} A_{i,2} \leq t'_{k-1,3} \qquad \text{---- (3)}$$

By Lemma 3.2; we have

$$t_{k,2} \leq t'_{k-1,3} \qquad \text{---- (4)}$$

Also, $t'_{k,2} = \max\left(y_k + \sum_{i=1}^{k} A_{i,2}, t_{k,2} \right)$

Using (3) and (4), we get

$$t'_{k,2} \le t'_{k-1,3}$$

Taking $k = n$, we have

$$t'_{n,2} \le t'_{n-1,3} \qquad \text{---- (5)}$$

Total time elapsed = $t_{n,3} = \max\left(t'_{n,2}, t'_{n-1,3}\right) + A_{n,3} = t'_{n-1,3} + A_{n,3} = t'_{n,3}$. (using 5)

Hence, the total time elapsed remains unaltered if M_2 starts processing jobs at time $L_2 = \min_{i \le k \le n} \{Y_k\}$.

5. Algorithm

The following algorithm is developed to find the latest time at which machines should be hired so as to minimize the total hiring cost for a fixed sequence of jobs processing.

Step 1: Prepare In-Out tables for S and compute total elapsed time $t_{n,3}(S)$.

Step 2: Compute the hiring time L_3 of M_3 for sequence S_k as

$$L_3(S) = t_{n3}(S) - \sum_{i=1}^{n} A_{i3}$$

Step 3: For the sequence S compute

I. $t_{n2}(S)$

II. $Y_1(S) = L_3(S) - A_{1,2}(S)$

III. $Y_q(S) = L_3(S) + \sum_{i=1}^{q-1} A_{i,3}(S) - \sum_{i=1}^{q} A_{i,2}(S); q = 2,3,......,n$

IV. $L_2(S) = \min_{1 \le q \le n} \{Y_q(S)\}$

V. $U_2(S) = t_{n,2}(S) - L_2(S)$.

Step 4: Compute total hiring cost of all the three machines for sequence S as:

$$R(S) = \sum_{i=1}^{n} A_{i,1} \times C_1 + U_2(S) \times C_2 + \sum_{i=1}^{n} A_{i,3} \times C_3.$$

6. Numerical Illustration

Consider a 5 jobs, 3 machines flowshop scheduling problem, the processing time of the machines are associated with probabilities and are as given in the table 1.The hiring cost per unit time of machine M_1,M_2 and M_3 are 8 units,6 units and 4 units respectively, with an objective to minimize the total hiring cost of the machines for the fixed sequence of jobs processing S: 1 – 3 – 2 – 5 – 4.

Table 1: The machines with processing time and corresponding probabilities

Jobs	Machine M_1		Machine M_2		Machine M_3	
i	a_{i1}	p_{i1}	a_{i2}	p_{i2}	a_{i3}	p_{i3}
1	18	0.2	8	0.2	20	0.2
2	14	0.3	10	0.3	12	0.1
3	16	0.2	12	0.2	13	0.3
4	32	0.1	11	0.2	10	0.1
5	20	0.2	13	0.1	15	0.3

Solution: The expected processing times are given in table 2.

Table 2: Expected processing times

Jobs (i)	M_1	M_2	M_3
	A_{i1}	A_{i2}	A_{i3}
1	3.6	1.6	4.0
2	4.2	3.0	1.2
3	3.2	2.4	3.9
4	3.2	2.2	1.0
5	4.0	1.3	4.5

The In – Out table for the fixed sequence S: 1 – 3 – 2 – 5 – 4 is

Table 3: The In-Out flow table for machines M_1, M_2 and M_3

Jobs	Machine M_1	Machine M_2	Machine M_3
i	In – Out	In – Out	In – Out
1	0.0 – 3.6	3.6 – 5.2	5.2 – 9.2
3	3.6 – 6.8	6.8 – 9.2	9.2 – 13.1
2	6.8 – 11.0	11.0 – 14.0	14.0 – 15.2
5	11.0 – 15.0	15.0 – 16.3	16.3 – 20.8
4	15.0 – 18.2	18.2 – 20.4	20.8 – 21.8

Total elapsed time $t_{n,3}(S) = 21.8$

Therefore, $L_3(S) = t_{n3}(S) - \sum_{i=1}^{n} A_{i,3}(S) = 21.8 - 14.6 = 7.2$

Also, for sequence S, we have $t_{n2}(S) = 20.4$

$Y_1 = 7.2 - 1.6 = 5.6, Y_2 = 7.2 + 4.0 - 4.0 = 7.2, Y_3 = 7.2 + 7.9 - 7.0 = 8.1, Y_4 = 7.2 + 9.1 - 8.3 = 8.0,$

$Y_5 = 7.2 + 13.6 - 10.5 = 10.3$

$L_2(S) = Min\{Y_k\} = 5.6,$

$U_2(S) = t_{n2}(S) - L_2(S) = 20.4 - 5.6 = 14.8$

The new reduced Bi-objective In – Out table is –

Table 4: The bi-objective In-Out flow table for machines M_1, M_2 and M_3 at latest times

Jobs	Machine M_1	Machine M_2	Machine M_3
i	In – Out	In – Out	In – Out
1	0.0 – 3.6	5.6 – 7.2	7.2 – 11.2
3	3.6 – 6.8	7.2 – 9.6	11.2 – 15.1
2	6.8 – 11.0	11.0 – 14.0	15.1 – 16.3
5	11.0 – 15.0	15.0 – 16.3	16.3 – 20.8
4	15.0 – 18.2	18.2 – 20.4	20.8 – 21.8

The latest possible time at which machine M_2 should be taken on rent = $L_2(S)$ = 5.6 units.
Also, utilization time of machine $M_2 = U_2(S)$ = 14.8 units.

Total Minimum hiring cost = $R(S) = \sum_{i=1}^{n} A_{i,1} \times C_1 + U_2(S) \times C_2 + \sum_{i=1}^{n} A_{i,3} \times C_3.$

$= 18.2 \times 6 + 14.8 \times 11 + 14.6 \times 7 = 374.2$ Units

Therefore the processing sequence *S*: 1 – 3 – 2 – 5 – 4 the minimum hiring cost is 374.2 units when M_1 starts processing job (.i.e. taken on rent) at time 0 units, M_2 at 5.6 units and M_3 at time 7.2 units.

Conclusion:

If M_3 starts processing jobs at time $L_3 = t_{n,3} - \sum_{i=1}^{n} A_{i,3}$ then the total elapsed time $t_{n,3}$ is not altered and M_3 is engaged for minimum time equal to sum of processing times of all the jobs on M_3, .i.e. reducing the idle time of M_3 to zero. If the machine M_2 is hired when it is required and is returned as soon as it completes the last job, the starting of processing of jobs at time

$$L_2(S) = \min_{1 \le q \le n}\{Y_q(S)\}, Y_1(S) = L_3(S) - A_{1,2}(S), Y_q(S) = L_3(S) + \sum_{i=1}^{q-1} A_{i,3}(S) - \sum_{i=1}^{q} A_{i,2}(S); \text{ q = 2, 3, …,n on } M_2$$

will, reduce the idle time of all jobs on it. Therefore total rental cost of M_2 will always be minimum. Also, the hiring cost of M_1 will also be minimum as the idle times of M_1 is always zero. The hiring cost for the machines when they start processing at their latest times is 374.2 units and the hiring cost from table 3, when they start processing as usual is 410.2 units. Therefore, the hiring cost is reduced. Hence, the proposed algorithm provides the decision makers a better idea when to hire machines. The study may be extended by introducing the concepts of independent setup time, transportation time, non availability constraints of machines for a certain interval of time.

References:

[1]. Anup (2002), 'On two machine flow shop problem in which processing time assume probabilities and there exists equivalent for an ordered job block', *JISSOR* ,Vol. XXIII, No. 1-4, pp. 41-44.

[2]. Bagga, P.C. (1969), 'Sequencing in a rental situations', *Journal of Canadian Operation Research Society*, Vol. 7 , pp.,152-153.

[3]. Chander, S., Rajendra, K. and Chanderi, D. (1992), 'An Efficient Heuristic Approach to the scheduling of jobs in a flow shop', *European Journal of Operation Research*, Vol. 61, pp. 318-325.

[4]. Gupta, J.N.D. (1976), 'Heuristic algorithms for the multistage flow shop problem', *AIIE Transactions*, Vol. 4, pp. 11-18.

[5]. Gupta, D., Singh, T.P. and Kumar, R. (2006), 'Minimizing rental cost in two stage flow shop, the processing time associated with probabilities including job block' *Reflection De Era*, Vol. 1, No. 2, pp. 107-120.

[6]. Gupta, D. and Sharma, S. (2011), 'Minimizing rental cost under specified rental policy in two stage flow shop, the processing time associated with probabilities including break-down interval and job – block criteria' , *European Journal of Business and Management*, Vol. 3, No. 2, pp. 85-103.

[7]. Ignall, E. and Scharge, L. (1965), 'Application of branch and bound technique to some flow shop scheduling problems', *Operation Research*, Vol. 13, pp. 400-412.

[8]. Johnson, S. M. (1954), 'Optimal two and three stage production schedule with set up times included', *Naval Research Logistic Quarterly*, Vol. 1, No.1, pp. 61- 68.

[9]. Narian, L. and Bagga, P.C. (2003), **'**Minimizing total elapsed time subject to zero total idle time of machines in n x 3 flowshop problem', *Indian J. pure appl. Math.*, Vol. 34, No. 2, pp. 219-228.

[10]. Narian, L. and Bagga, P.C. (2005), 'Flowshop / No-idle scheduling to minimize the mean flowtime' *ANZIAM J.*, Vol. 47, No. 2005, pp. 265-275.

[11]. Narian, L. and Bagga, P.C. (2005**), '**Scheduling problems in Rental Situation', *Bulletin of Pure and Applied Sciences: Section E. Mathematics and Statistics*, 24, ISSN: 0970-6577.

[12]. Narain, L. (2006), 'Special models in flow shop sequencing problem', *Ph.D. Thesis, University of Delhi*, Delhi.

[13]. Singh, T.P. (1985), 'On n x 2 shop problem involving job block, transportation times and breakdown machine times' *Pure and Applied Mathematika Sciences*, Vol. XXI, No. 1985, pp. 1-2.

[14]. Szware, W. (1977), 'Special cases of the flow shop problem*s', Naval Research Logistic, Quarterly*, Vol. 24, No. 1977, pp. 403-492.

[15]. Yoshida and Hitomi (1979**), '**O*ptimal* two stage production scheduling with set up times separated', *AIIE Transactions*, Vol. II, pp. 261-263.